

Inflation and Monetary Policy

In this problem, we consider how a central bank will try and use monetary policy to control inflation and output in the macroeconomy.

Consider an economy whose rate of growth at (discrete) time n is given by $\bar{y} + y_n$, and let the inflation rate during this time period be π_n . Here $\bar{y}$ is the average rate of growth, and y_n represents deviations from this behavior. On the average, we would like to expect $\langle y_n \rangle = \langle \pi_n \rangle = 0$.

The basic rule of thumb for the central bank is that when inflation gets high, the bank needs to try and lower the rate of output to stop the economy from overheating. Let us assume the simplest possible behavior for the central bank: given the knowledge at time n , the bank will alter the rate of growth so that

$$\langle y_{n+1}|n \rangle = -q \langle \pi_{n+1}|n \rangle.$$

The central bank wishes to minimize a parameter F_n , which is defined as

$$F_n = \left\langle \lambda \pi_n^2 + (y_n + \bar{y} - \hat{y})^2 \right\rangle$$

where λ is a parameter describing the relative weight the bank puts on inflation to output, and $\hat{y}$ is the preferred output. Finally, let us assume that

$$\begin{aligned} y_{n+1} &= c y_n - b r_n + z_n \\ \pi_{n+1} &= \pi_n + \alpha y_n + \rho_n \end{aligned}$$

where $0 < c < 1$, $b > 0$, z_n are i.i.d. Gaussian random variables with zero mean and variance σ^2 , $\alpha > 0$ is a parameter, and ρ_n are i.i.d. Gaussian random variables with zero mean and variance τ^2 . z_n and ρ_n are also independent of y_t and π_t for $t \leq n$. r_n is the variable chosen by the central bank to ensure the that $\langle y_{n+1}|n \rangle = -q \langle \pi_{n+1}|n \rangle$.

- (a) Find an expression for $\langle \pi_n^2 \rangle$ in terms of α , q , σ and τ , in the limit of $n \rightarrow \infty$. To do this, use the fact that this should converge to an equilibrium distribution.
- (b) Using this result, show that the optimal choice of q is given by

$$q^* = \frac{\sqrt{\alpha^2 \lambda^2 + 4\lambda} - \lambda \alpha}{2}.$$

- (c) What is the asymptotic behavior of q^* when $\lambda \gg \alpha$? What does this behavior of λ imply that the central bank cares about?
- (d) In this regime, compute $\langle \pi_{n+2}|n \rangle$, and comment on the result.