Heating a Room

A heater is turned on in a room, openly connected to the outside, so that the room maintains a constant pressure P and volume V. The following constants are known for the air:

$$\alpha = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_{P,N},$$

$$c_{\mathrm{P}} = \frac{1}{N}\left.\frac{\partial E}{\partial T}\right|_{P,N}.$$

(a) Show that

$$\left.\frac{\partial S}{\partial T}\right|_{P,V} = -\alpha S + \frac{N c_{\mathrm{P}}}{T}.$$

This is tricky: to get started, consider writing $\partial_T = (\partial_T N)\partial_N$. Then, use the Gibbs-Duhem relation for chemical potential, and more partial derivative tricks.

(b) Show that if $H = E + PV$ is the enthalpy:

$$\left.\frac{\partial E}{\partial T}\right|_{P,V} = -\alpha H + N c_{\mathrm{P}}.$$

(c) Evaluate the results of part (a) and (b) for a monoatomic ideal gas, and show that the room is heated by decreasing entropy, not increasing energy.