

Life at the End of the Universe

If we live in an open universe, which is slowly cooling down as it expands, and stars die off, etc., what will happen to life? Assume that an intelligent being needs to come up with a method to use a finite amount of energy and be able to have an infinite number of thoughts. This is equivalent to infinitely increasing the amount of entropy of the universe. Define

$$U(t) = \int_0^t dt' h(t')T(t'),$$

where $h(t)$ is a measure of the percentage of time the creature is spent hibernating (using up effectively no energy), and $T(t)$ is the temperature of the creature. We hypothesize that this is the *effective time* that our creature has lived, in that the number of thoughts Θ generated by the creature is proportional to

$$\Theta = \eta U$$

for some constant η . Let s be the entropy per thought generated by the creature: take it as a constant.

(a) Show that the metabolic rate of this creature is given by

$$P_m(t) = s\eta T(t)^2 h(t).$$

If the creature has N electrons of mass m and charge $-e$ which generate thoughts by moving about and emitting electric dipole radiation, it can be shown that there is a limit on the power radiated, using quantum mechanics:

$$P_r \leq \frac{\gamma N e^2}{2\pi m} T^3$$

where γ is some $O(1)$ constant.

(b) Use the fact that $P_r \geq P_m$ to show that

$$T(t) \geq \text{constant} \times h(t).$$

Ultimately, the creature must spend more and more of his time hibernating.

Now, assume that the creature can pretty much maintain himself at the minimum temperature possible, which is the background temperature of the universe. Cosmology tells us that this minimum temperature is

$$T(t) = \frac{\alpha}{t^{1/3}}.$$

(c) Show that the choice

$$h(t) = \frac{\beta}{t^{3/8}}$$

allows the being to have an infinite number of thoughts while only using a finite amount of energy W . Find an expression for W .