Ocean of the Early Earth

As early as 4.2 billion years ago, we know that there was liquid water on the surface of the Earth. Does this provide strong lower bounds on the temperature of the primordial Earth?

(a) Argue that if the dominant gas in the atmosphere, which is a thin shell of vapor around the Earth, is water vapor, then the mass of the water vapor in the atmosphere is given by

$$M_{\mathrm{v}} = \frac{4\pi R^2 P}{g},$$

if g is the gravitational field at the surface of the Earth, P is the pressure at the surface of the Earth, and R is the radius of the Earth.

(b) You can approximate that the saturation pressure of water in the ocean is

$$P_{\mathrm{sat}}(T) = P_0 \exp\left[-\frac{L}{k_{\mathrm{B}}(T+t)}\right].$$

The parameters of interest are $g \approx 10$ m/s^2, $R \approx 6 \times 10^6$ m, $P_0 \approx 10^{10}$ Pa, $L \approx 4 \times 10^{-18}$ J, and $t \approx 230$ K. If the total mass of water on the Earth was 10^{21} kg, what is an upper bound on the temperature of the primordial Earth?

(c) Show that there is a critical ocean mass M_{c} at which the ocean will never fully evaporate, no matter how hot the temperature is. Find the expression for M_{c}, and evaluate it for the parameters above. Is this a likely outcome?