Strangelet Matter

One of the (incredibly unlikely) doomsday scenarios about colliders such as the RHIC heavy ion collider is the possible formation of strangelet matter: a giant ball of strangelet matter, which is proposed to be thermodynamically preferred to normal matter, could then suck up all matter on Earth, ending life on the planet.

Strangelet matter is essentially a soup of up, down and strange quarks and electrons that may be a preferred thermodynamic phase to our normal state of nuclear matter, which consists of normal protons and neutrons. The main idea is that, although strange quarks are very heavy, in a large quark soup, it is preferable to have strange quarks because the Pauli exclusion principle means that these quarks can be at lower momenta. We will explore this possibility in a simple quantitative model, in this problem.

Let us begin with a brief overview of the relevant nuclear physics. We will be dealing with electrons, which can exist in 2 distinct spin states. Quarks are also spin-1/2 particles with 2 spin states – in addition, they can also exist in 3 different color states. For this problem, we will set $\hbar = c = 1$, so masses, energies and momenta are all measured in the same units, which we'll take to be MeV. We denote the mass of the strange quark with m. The masses of electrons (e), up quarks (u) and down quarks (d) are very small in comparison, and we will take them to be approximately 0. If the electric charge of the electron is $-e$, then the electric charge of u is $2e/3$, and the charges of d and s are $-e/3$.

Now, we turn to strangelet matter. Define the chemical potentials for our 4 species as $\mu_{\mathrm{e,u,d,s}}$ (labels are self-evident). To first approximation, let us just treat the strangelet matter as consisting of 4 Fermi gases of non-interacting particles. For each species of particle, we can define a grand thermodynamic potential density ω:

$$\omega \equiv \epsilon - \mu n,$$

where n is the number density of the species, and ϵ is the energy density.

(a) Show that

$$\begin{aligned}
\omega_{\mathrm{u,d}} &= -\frac{\mu_{\mathrm{u,d}}^4}{4\pi^2}, \\
\omega_{\mathrm{e}} &= -\frac{\mu_{\mathrm{e}}^4}{12\pi^2}, \\
\omega_{\mathrm{s}} &= -\frac{1}{4\pi^2}\left[\mu_{\mathrm{s}}\left(\mu_{\mathrm{s}}^2 - \frac{5}{2}m^2\right)\sqrt{\mu_{\mathrm{s}}^2 - m^2} + \frac{3}{2}m^4 \log\frac{\mu_{\mathrm{s}} + \sqrt{\mu_{\mathrm{s}}^2 - m^2}}{m}\right].
\end{aligned}$$

The last expression is exact, but find asymptotic expressions for it when $\mu \gg m$, and when $\mu - m \ll m$.

(b) The bulk system must have no net electric charge, and in addition we would like the following reactions[1] to be in equilibrium:

$$\begin{aligned}
\mathrm{d} &\rightleftharpoons \mathrm{u} + \mathrm{e}, \\
\mathrm{s} &\rightleftharpoons \mathrm{u} + \mathrm{e}, \\
\mathrm{d} + \mathrm{u} &\rightleftharpoons \mathrm{s} + \mathrm{u}.
\end{aligned}$$

[1] For simplicity, we have ignored the neutrions present in these reactions, as they will not be treated in our thermodynamic approximation.

Conclude that there is only one independent chemical potential – let us choose to pick $\mu_{\rm s} \equiv \mu$ as our lone chemical potential for the remainder of the problem.

(c) Study the number densities of each of the 4 species in the asymptotic regimes identified above, and comment on the results.

(d) For the system to be in mechanical equilibrium with the vacuum, the pressure of the strangelet matter must be equal to the vacuum pressure – 0. Suppose that there is an energy density of B associated with the vacuum of strangelet matter – this is related to the confining interactions between quarks. Conclude that

$$0 = B + \omega_{\rm e} + \omega_{\rm u} + \omega_{\rm d} + \omega_{\rm s},$$

and therefore that $B > 0$. Estimate the value of B associated to the strangelet phase in the asymptotic regimes of μ.

(e) Now, we need to check the energy per baryon, $h \equiv \epsilon/(n_{\rm u} + n_{\rm d} + n_{\rm s})$: if $h > 940$ MeV, which is the energy of a nucleon, then strange matter will be unstable to the emission of nucleons. Using $m \approx 100$ MeV, determine numerically the allowed range of B for which strangelet matter is stable.

(f) Now, consider the energy per baryon of non-strange matter, where there are no strange quarks, and instead a simple quark soup of u, d and e. In this case, determine numerically the value of B for which the energy per baryon is less than 930 MeV – in this regime, this simple quark matter is also stable. Show that there is a parameter regime in B for which *only* the strangelet matter is stable.

So far, no strangelet matter has been observed experimentally. In this problem, we have neglected quantum corrections to ωs, but it is possible in the so-called strong coupling limit that the electric charge due to d and s alone could make the strangelet matter negatively charged – then, positrons would compensate for the overall charge density. If this happened, then ordinary atoms would be attracted to the negatively charged object and become absorbed into it. This is essentially the doomsday scenario behind strangelet matter.