

Waves in Cubic Crystals

The free energy density of an elastic crystal with mass density ρ and cubic symmetry may be written in terms of the strain tensor s_{ij} in the following form:

$$\mathcal{F} = \frac{\rho}{2} \left[a s_{ij} s_{ij} + b (s_{kk})^2 + c (s_{xx}^2 + s_{yy}^2 + s_{zz}^2) \right]$$

- (a) Write down the equation of motion for u_x (the other two will look very similar, of course!).
- (b) Consider a plane wave of the form $e^{ik(x \cos \theta + y \sin \theta) - i\omega t}$ propagating through the crystal. Find all distinct modes of propagation associated to this plane wave.
- (c) Draw sample wavefronts propagating through the crystal after an arbitrary displacement occurs. Be sure to consider both $c > 0$ and $c < 0$.