continuum mechanics  $\rightarrow$  wave propagation

## **Jeans Instability**

A non-viscous fluid fills the interstellar medium. In equilibrium the fluid has a speed of sound c and a mass density  $\rho_0$ . Now, assume a slight perturbation to this static configuration, and assume that the perturbations in the fluid density create gravitational fields through Newtonian gravitation.

- (a) Write down the Poisson equation for the gravitational potential  $\varphi$ , as well the dynamical wave equation for the fluid (what would be a convenient variable to work with?).
- (b) Show that the dispersion relation for sound waves is modified to

$$\omega^2 = c^2 k^2 - 4\pi G \rho_0.$$

- (c) Given  $G \approx 7 \times 10^{-11} \text{ N/kg}^2 \cdot \text{s}^2$ ,  $\rho_0 \approx 10^{-21} \text{ kg/m}^3$  and  $c \approx 400 \text{ m/s}$  for interstellar dust, find the critical value of  $k_c$  at which an instability occurs. This instability is called the **Jeans instability**.
- (d) Do you think any interstellar structures could be formed by the Jeans instability?<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>For relevant length scales from astronomy, Wikipedia will be a good source!