Rayleigh-Taylor Instability

A fluid of mass density ρ_2 extends in equilibrium for $z > 0$, and one of mass density ρ_1 extends for $z < 0$. Both are assumed to be inviscid and incompressible. Suppose that waves of the form $\mathrm{e}^{\mathrm{i}(k_x x + k_y y - \omega t)}$ are propagating through both fluids, displacing the boundary between the fluids to $\zeta(x, y, t)$. (Assume ζ is "small").

(a) Surface tension at the boundary can be described by

$$\Delta P = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

Show that for small ζ, the above equation is equivalent to

$$\Delta P = -\gamma k^2 \zeta \delta(z)$$

where $k^2 = {k_x}^2 + {k_y}^2$.

(b) Using conservation of mass and momentum, show that the dispersion relation for surface waves along the boundary is

$$\omega^2 = \frac{k(\gamma k^2 - g(\rho_2 - \rho_1))}{\rho_2 + \rho_1}.$$

(c) In what regimes is there instability? Give a physical justification for the answer.

(d) When an instability exists, find the largest rate of growth of an instability.